

## **A Mixed Bi-directional S-curve Acceleration/Deceleration Control Algorithm for Continuous Segments**

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**Abstract:** *During the continuous segments interpolation for high-speed machining, the 7-period S-curve Acceleration/Deceleration(ACC/DEC) algorithm is complicated to implement because of complex classification and huge computation. Though 5-period S-curve is a simplified model, it spends more time in accelerating and decelerating because acceleration changes constantly. Therefore a mixed bi-directional S-curve ACC/DEC interpolation algorithm is presented to control the velocity from the positive and reverse of each segment, and obtain the information of feedrate and displacement. It can significantly reduce the time of velocity planning, improve the machining efficiency, and effectively avoid the mechanical vibration caused by the saltation of velocity and acceleration. Finally, the proposed algorithm is analyzed and validated by using a simulation example.*

**Keywords** - *High-speed machining, S-curve algorithm, acceleration/deceleration control, interpolation,*

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### **I. INTRODUCTION**

High-speed and high-precision numerical control (NC) machining is mainly applied to machining a series of continuous line segments that are discretely formed by complex curves and surfaces [1]. When NC interpolates the continuous line segments, the ACC/DEC algorithm is widely used as follows: linear ACC/DEC algorithm [2], exponential ACC/DEC method [3], S-curve ACC/DEC algorithm [4]. Due to the easy formulation and less computation, linear ACC/DEC algorithm and exponential ACC/DEC method are generally used in economical computer numerical control (CNC) systems. However, the acceleration mutation in two ACC/DEC algorithms causes machine tool vibration and affects the machining quality. S-curve ACC/DEC algorithm has good flexibility and high machining quality, and can realize the continuous smooth changes of velocity and acceleration, which becomes one of the most important research in ACC/DEC algorithm and is widely used in high-grade CNC systems [5].

S-curve ACC/DEC algorithm can be divided into 7-period S-curve and 5-period S-curve, and 5-period S-curve is obtained from the 7-period S-curve which simplifies the uniform acceleration and the uniform deceleration. Mecklet et al. [6] proposed a method to minimize the residual vibration by optimizing the parameters of asymmetric S-curve feedrate profile. Erkorkmaz et al. [7] presented a S-curve algorithm by generating a quintic spline trajectory to produce continuous position, velocity, and acceleration profiles. Chen et al. [8] developed an S-curve ACC/DEC algorithm to generate feedrate profiles with limited values of feedrate, feed acceleration, and feed jerk by using a quintic feedrate function. Yau et al. [9] proposed a method to make the single block achieve  $C^1$  continuity and jerk-limited capability by using the S-shaped feedrate profile for ACC/DEC planning. Zhang et al. [10] used the equivalent trapezoidal velocity profile to analyze the speed of S-curve velocity profile and work out the accurate interpolation method of S-curve velocity profile. Zheng et al. [11] proposed the 5-period S-curve to implement S-curve acceleration and deceleration based on the control of anticipation time. The

7-period S-curve algorithm is complicated to implement because of complicated classification and huge computation. Compared to 7-period S-curve, the 5-period S-curve is a simplified model, but its acceleration changes constantly and cannot maintain a higher value so that the velocity changes slower and spends more time in accelerating and decelerating.

In this paper a 7-period and 5-period mixed bi-directional S-curve ACC/DEC algorithm is proposed to control the velocity from the positive and reverse of each segment and obtain the information of velocity and displacement. Due to the symmetry between the acceleration period and the deceleration period in S-curve, the deceleration period can be regarded as the reverse acceleration period. Thus only the acceleration period is considered in S-curve algorithm so as to simplify the operation. So the proposed algorithm can significantly reduce the time of velocity planning, improve the machining efficiency, and achieve the continuous smooth of velocity and acceleration.

The rest of this paper is organized as follows: the mixed bi-directional S-curve ACC/DEC algorithm is presented in Section 2, which including the basic principle of the algorithm, the determination of velocity parameters of the acceleration region, and the determination of the mixed S-curve type and parameters. Then Section 3 presents the simulation and experimental results obtained with the proposed algorithm, and the conclusions are summarized in Section 4.

## **II. A mixed bi-directional S-curve ACC/DEC algorithm**

### **2.1 The basic principle of the algorithm**

Fig. 1 presents a mixed bi-directional ACC/DEC algorithm with 7-period S-curve and 5-period S-curve. The basic idea is that if the CNC accelerates from the initial velocity (transition velocity)  $v_i$  and end velocity  $v_{i+1}$  to the maximum velocity  $v_{max}$  along the positive and reverse of the (i+1)th tool path respectively, then the type of the S-curve ACC/DEC algorithm can be obtained by comparing the relations between displacement  $S_a$  of the positive acceleration period, the displacement  $S_b$  of the reverse acceleration period and the (i+1)th displacement  $S$ . In practical applications, it is difficult for the CNC system to predefine whether the machining velocity  $v$  can reach the programming velocity  $v_f$  so that the calculation formula of S-curve cannot be determined. So by assuming the machining velocity can reach the programming velocity, and then the type of the S-curve ACC/DEC algorithm and the corresponding calculation equations are decided by the comparison of displacement and velocity.

The type of S-curve ACC/DEC algorithm can be judged in Fig. 2. Where ① shows that the velocity can reach the programming velocity  $v_f$  along the positive and reverse tool path respectively, and the S-curve has the uniform period. ② represents that if  $S_a + S_b > S$ , then the velocity cannot reach the programming velocity  $v_f$ , and the velocity parameters can be recalculated by setting  $S_a + S_b = S$ , and the S-curve has the acceleration period and deceleration period. The S-curve only has the uniform period in ③. ④ shows that if  $S_a < S$ , then the type of S-curve is acceleration period and uniform period. If  $S_a = S$ , then the S-curve only has the acceleration period. If  $S_a > S$ , then setting  $S_a = S$  and the S-curve only has the acceleration period, therefore velocity can be calculated. ⑤ represents that if  $S_b < S$ , then the type of S-curve is uniform period and deceleration period. If  $S_b = S$ , then the S-curve only has the deceleration period. If  $S_b > S$ , then setting  $S_a = S$ , so velocity parameters can be worked out because the S-curve only has the reverse acceleration period. ⑥ shows that the velocity can just accelerate to  $v_f$  along the positive and reverse tool path respectively, so that

the S-curve only has the acceleration period and deceleration period.

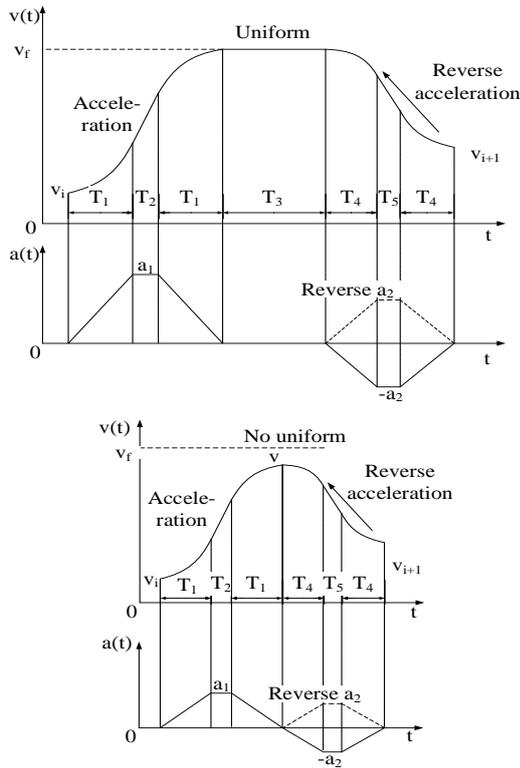


Fig. 1. S-curve ACC/DEC

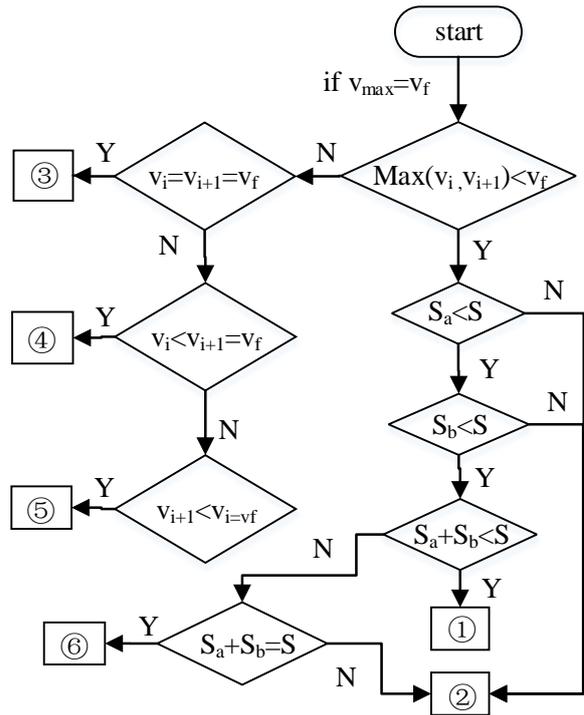


Fig. 2. The flow chart for determining the type of S-curve ACC/DEC algorithm

### 2.2 The determination of velocity parameters of the acceleration period

Taking the positive ACC-period as example, and  $J$  is the jerk, and the variation of the velocity  $\Delta v$  is computed as  $\Delta v = v - v_i$  ( $\Delta v$  reaches to the maximum value when  $v$  is equal to  $v_f$ ). Due to the different formulae of displacement between 7-period ACC/DEC and 5-period ACC/DEC, the critical of the mixed ACC/DEC algorithm is to determine displacement formula. By considering the ACC/DEC formula of S-curve algorithm, it is concluded that the variation of the velocity for 5-period ACC/DEC is obtained by  $\Delta v = a^2 / J$ , while the velocity changes in 7-period ACC/DEC can be expressed as  $\Delta v > a^2 / J$ . By comparing  $\Delta v$  and  $a^2 / J$ , the calculation formula of displacement can be determined as:

$$S_a = \begin{cases} (v + v_i)\sqrt{\Delta v / J} & \Delta v \leq a_{max}^2 / J \\ \frac{(v + v_i)(J\Delta v + a_{max}^2)}{2Ja_{max}} & \Delta v > a_{max}^2 / J \end{cases} \quad (1)$$

The concrete algorithm is as follows:

Step 1. After calculating  $\Delta v$  and  $a_{max}^2 / J$ , the corresponding displacement formula can be computed by using Eq. (1).

Step 2. If  $\Delta v \leq a_{max}^2 / J$ , and the displacement  $S_a$  of acceleration period can be obtained. In order to guarantee the acceleration and jerk not to exceed the kinematical limits of machine tool, the acceleration  $a$  is adjusted to  $a = \sqrt{J\Delta v}$  and the time of acceleration and deceleration can be computed as  $T_1 = a / J$ .

Step 3. If  $\Delta v > a_{max}^2 / J$ , after acquiring the acceleration displacement  $S_a$ , then the parameters can be solved according to the formula of 7-period S-curve, and the time of acceleration period and deceleration period is calculated by  $T_1 = a_{max} / J$ , and the acceleration time  $T_2$  is expressed as  $T_2 = \frac{\Delta v}{a_{max}} - \frac{a_{max}}{J} = \frac{\Delta v}{a_{max}} - T_1$ .

#### 2.4 The determination of the mixed S-curve type and parameters

One reason why S-curve algorithm computes complexly is that different lengths of machining paths lead to different calculation formulae of S-curve. If the path length is long enough, the machining velocity can reach the programming velocity, so that there is uniform period in the S-curve. If the path length is short, the machining velocity is less than the programming velocity, thus the S-curve does not have uniform period. Hence the determination of the mixed S-curve type and parameters must be divided into two cases.

##### (1) The machining velocity can reach the programming velocity

In this case, the velocity increases from  $v_i$  to  $v_f$ , and keeps uniform  $v_f$ , then decreases to  $v_{i+1}$ . Where  $S$  is the machining path length,  $S_a$  is the length of acceleration period,  $S_b$  is the length of reverse acceleration period. When the machining velocity reaches the programming velocity  $v_f$ , the displacement must satisfy that  $S_a + S_b \leq S$  (when  $S_a + S_b = S$ , the machining velocity can reach the programming velocity  $v_f$ , while there is no uniform period). So we can obtain  $\Delta v = v_f - v_i$ ,  $\Delta v = v_f - v_{i+1}$  and  $a_{max}^2 / J$ , then the time, displacement, etc. and the type of S-curve can be determined by using the algorithm in section 2.2. When  $S_a + S_b < S$ , there is uniform period in the S-curve, so the length of uniform period and the uniform time can be formulated as  $S_c = S - S_a - S_b$  and  $T_3 = (S - S_a - S_b) / v_f$ , respectively.

##### (2) The machining velocity cannot reach the programming velocity

There is  $S_a + S_b > S$  when the machining velocity cannot reach the programming velocity. So only the maximum machining velocity is smaller than  $v_f$ , we can conclude that  $S_a + S_b = S$ . Thus the S-curve has no uniform period, and for  $S_a + S_b = S$ , the velocity parameters must be recalculated. Because the unknown of the maximum velocity leads to the uncertain relationship between the  $\Delta v$  and  $a^2 / J$ , there are several possible calculation formulae of displacement. In order to simplify calculation and definite displacement formulae, supposing the relation between  $\Delta v$  and  $a^2 / J$  is just satisfy with the 5-period S-curve, then the variety of velocity in acceleration and deceleration can be expressed as:

$$\begin{cases} v - v_i = a_{max}^2 / J_1 \\ v - v_{i+1} = a_{max}^2 / J_2 \end{cases} \quad (2)$$

The displacements are calculated as follows:

$$\begin{cases} S_a = (v + v_i) a_{max} / J_1 \\ S_b = (v + v_{i+1}) a_{max} / J_2 \\ S_a + S_b = S \end{cases} \quad (3)$$

The maximum machining velocity is solved as  $v = \sqrt{(S a_{max}^2 + v_i^2 + v_{i+1}^2) / 2}$  from Eq. (2) and Eq. (3), then the

jerks  $J_1$ ,  $J_2$ , and time  $T_1$ ,  $T_4$  can be obtained. Thus, we know all the parameters of the mixedbi-directional S-curve ACC/DEC algorithm, then the formulas of the displacement, velocity, acceleration can be written.

### III. Simulation and experiment results

In this section, the proposed algorithm is proved to be correct and efficient by the experimental segments showed in Fig. 3 based on MATLAB. NC machining parameters are used the following settings: the programming velocity  $v_f$  is constrained to 60 mm/s, the maximum acceleration  $a_{max}$  is 3000 mm/s<sup>2</sup>, the jerk is limited to 200 m/s<sup>3</sup>. And the interpolation period  $T$ , the starting velocity, and the ending velocity are set to 4 ms, 0 and 0, respectively.

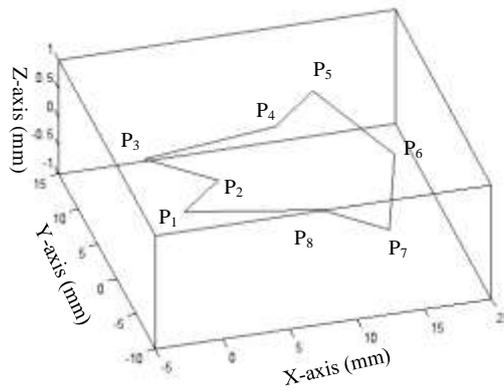


Fig. 3 Experimental segments

Tab. 1 The summarized motion results.

Path segments	1	2	3	4	5	6	7	8
$v_i$ (mm/s)	8.5	6.3	19.4	7	19.4	7	27.8	0
$S_i$ (mm)	5	6	10.4	5.5	10.4	10.4	5.5	9.86
$S_a$ (mm)	1.1	1.1	1.1	1.13	1.09	1.13	1.09	1.13
$S_b$ (mm)	1.1	1.1	1.13	1.1	1.13	1.09	1.13	1.1
$S_c$ (mm)	2.8	3.8	8.17	3.27	8.18	8.18	3.28	7.63

Tab.2 Computational time

Computational time (s)	Look-ahead pre-interpolation	Interpolation
The proposed algorithm	0.022	1.25
5-period S-curve	0.038	1.293

Tab. 1 lists the summarized algorithm results and this paper takes the third segment as an example to illustrate the effectiveness of the proposed algorithm

Firstly, the transition velocity  $v_3$  and path length  $S_3$  of the third segment can be obtained with  $v_3 = 19.4 \text{ mm/s}$  and  $S_3 = 10.4 \text{ mm}$ . Secondly, according to Eq. (8), for positive acceleration period, we can

calculate that  $\Delta v = 53.7 > a_{max}^2 / J = 45$ , and compute the acceleration displacement  $S_a$  with 1.1 mm. For reverse acceleration period, there is  $\Delta v = 40.6 < a_{max}^2 / J = 45$ , and we can obtain the reverse acceleration displacement  $S_b$  with 1.13 mm. So we can conclude  $S_a + S_b < S_3$ , thus the type of the third segment can be determined as type ① which the velocity can reach the maximum velocity  $v_f$  along the positive and reverse respectively, and the S-curve has the uniform period. Similarly we can verify the other paths, this paper will not be discussed in detail.

After the implement of MALAB programming, the velocity profile and acceleration profile which can be obtained by comparing the proposed algorithm and 5 S-curve algorithm which are shown in Fig. 4 and Fig. 5.

As can be seen in Fig. 4 and Fig. 5, the proposed algorithm and 5-period S-curve algorithm can both realize the velocity and acceleration transition continuously and smoothly, and improve the flexibility of controlling system. Thus can effectively avoid the tool vibration and overcut caused by acceleration and velocity mutation, and greatly improve the machining quality of parts. The two algorithm are computed with Intel i5-4210m CPU and 8G RAM. As shown in Table 2, the machining time of proposed algorithm is 1.254s, compared with the operation time 1.293s of 5-period S-curve, the working time is shortened by 0.039s. Specially, the proposed algorithm is very useful for the look-ahead interpolation period. It is faster nearly twice time than the 5-period S-curve algorithm. Furthermore, with the increase of machining paths, the working efficiency will be improvement more obvious. Therefore, the proposed algorithm has faster response time, better real-time performance, and more efficiency.

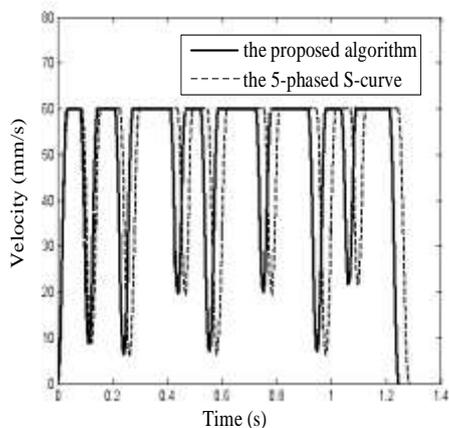


Fig. 4 Comparison of velocity

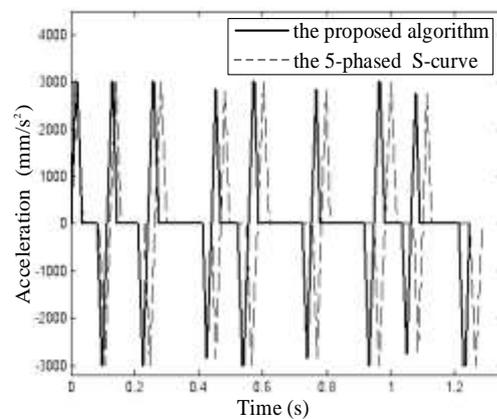


Fig. 5 Comparison of acceleration

#### IV. Conclusions

One focal and difficult point for high-speed machining of continuous segments is on basis of simplicity and rapidity trying to guarantee high-speed and achieve the velocity and acceleration transition continuously and smoothly. Hence, based on the 7-period S-curve and 5-period S-curve, this paper proposes a 7-period and 5-period mixed bi-directional S-curve ACC/DEC Algorithm to plan the velocity from positive and reverse of each path, and obtain the information of speed and displacement. The proposed algorithm can guarantee the simplicity of calculations, significantly reduce the time of velocity planning, improve the machining efficiency, and realize the continuous smooth changes of velocity and acceleration.

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## References

- [1] Ye Peiqing, Shi Chuan, Yang Kaiming, et al. Interpolation of continuous micro line segment trajectories based on look-ahead algorithm in high-speed machining[J]. *Int J AdvMnuufTechnol*, 2008, 37(9-10): 881-889.
- [2] WangYuhan, XiaoLingjian, ZengShuisheng, et al. An optimal feedrate model and solution for high-speed machining of small line blocks with look-ahead[J]. *Journal of Shanghai Jiaotong University*, 2004, 38(6): 901-904.
- [3] Xu Chuangwen. Study of accelerating and decelerating control on NC feed system[J]. *Chinese Journal of Scientific Instrument*, 2002, 23(Suppl. 3): 360-362.
- [4] Tao Jia-an, Gao Shenglin, You Qingning, et al. Lookahead and Acceleration/Deceleration Algorithms for Micro-Line Blocks Machining[J]. *Journal of Computer-Aided Design & Computer Graphics*, 2010, 22(9): 1570-1577.
- [5] Zhu Ming, You Youpeng, HE Jun. Research on S-shape Acceleration/Deceleration Algorithm in Look-ahead[J]. *Journal of Chinese Computer Systems*, 2011, 32(10): 2140-2144.
- [6] Meckl P.H, Arestides P.B, Optimized S-curve motion profiles for minimum residual vibration[J]. *Proceedings of the 1998 American Control Conference*, 1998, 2627-2631.
- [7] Erkorkmaz K, Altintas Y. High speed CNC system design. Part I: jerk limited trajectory generation and quintic spline interpolation[J]. *International Journal of Machine Tools & Manufacture* 41 (2001) 1323–1345.
- [8] Jin-Hung Chen, Syh-ShiuhYeh, Jin-Tsu Sun. An S-curve Acceleration/Deceleration Design for CNC Machine Tools Using QuinticFeedrateFunction[J]. *Computer-Aided Design and Applications*, 8:4, 583-592
- [9] Yau H.-T, Wang J.-B, Hsu C.-Y, Yeh C.-H. PC-based controller with real-time look-ahead NURBS interpolator[J]. *Computer-Aided Design and Applications*, 4(1-6), 2007, 331-340.
- [10] Zhang Deli, ZhouLaishui. Adaptive algorithm for feedrate smoothing of high speed machining[J]. *ActaAeronauticaetAstronauticaSinica*, 2006, 27(1): 125-130.
- [11] ZhengZhongqian, WangXingfei, Li Song, Li Di. Algorithm of S-shape acceleration based on control of anticipation time[J]. *Journal of Mechanical & Electrical Engineering*, 2014, 31(4): 425-430.

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